Supplementary material for
"Blood flow velocity field estimation via spatial regression with PDE penalization"

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1 Simulation studies with areal observations

In this supplementary material we report simulation studies with areal observations that mimic our application setting. Since SOAP is not currently implemented for areal observations, SR-PDE estimates are compared only to SSR estimates. Specifically, we consider two cases:

D. N=21 subdomains $D_1, \ldots, D_N$ uniformly distributed on the entire domain Ω;

E. N=7 subdomains in the same cross-shape pattern used in the application study.

The domain Ω and the true function f_0 are the same used in the previous simulation studies and displayed in Figure 7 of the main paper. The upper left panel of the figures shows the subdomains considered in each of the two different scenarios. Each subdomain is colored according to the spatial average of the true surface f_0 computed on the same subdomain. The same color scale of Figure 7 of the main paper is used. The experiment is replicated 50 times. For each study case, D and E, and each replicate: we sample independent errors, $\eta_1, \ldots, \eta_N$, from a Gaussian distribution with mean 0 and standard deviation $\bar{\sigma} = 0.05$, and we thus obtain observations $\bar{z}_1, \ldots, \bar{z}_N$ according to model (7) introduced in the main paper. The details for SR-PDE and SSR models are the same as in Section 6 of the main paper, with the only difference that the smoothing parameter λ is not chosen via cross-validation, due to the small number of observations. The parameter λ is

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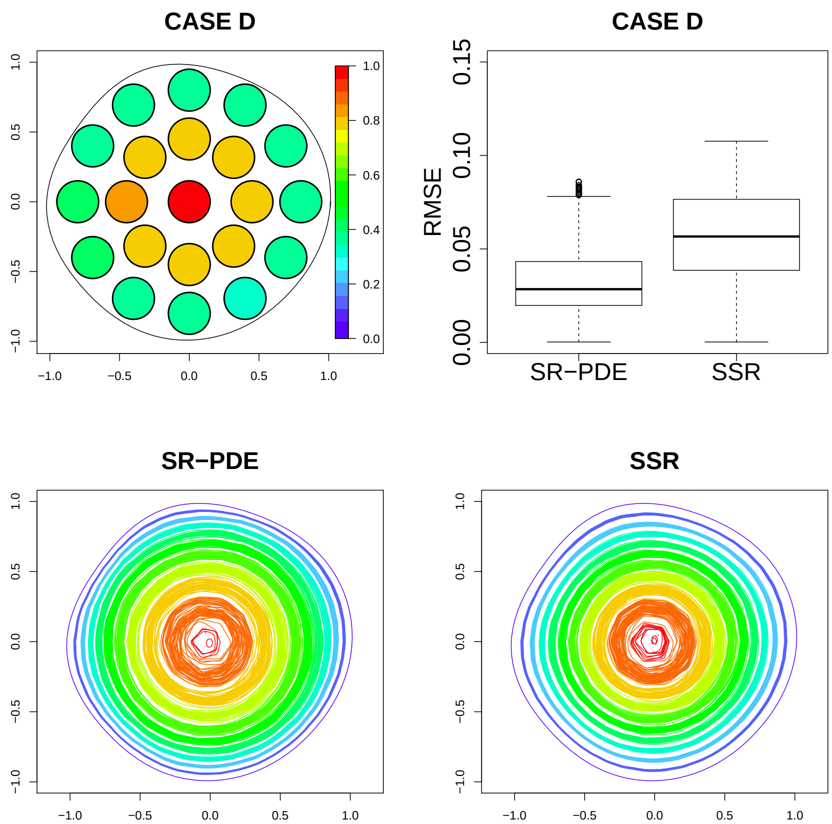

Figure 1: Top left: subdomains considered in case D; each subdomain is colored according to the spatial average of the real surface f_0 on the subdomain. Top right: boxplot of RMSE (evaluated on a fine lattice of step 0.01 over the domain Ω) for SR-PDE and SSR estimators. Bottom left, bottom right: surface estimates obtained using respectively SR-PDE and SSR; the images display the isolines $(0, 0.1, \ldots, 0.9, 1)$ of the surface estimates obtained in the 50 simulation replicates; the isolines are colored using the same color scale used for the isolines of the true function f_0 in Figure 7 of the main paper.

instead fixed to $\lambda = 10^{-1}$ for the anisotropic and non-stationary SR-PDE model and $\lambda = 10^{-3}$ for the isotropic and stationary SSR model. These values are chosen graphically on a grid of values in order to obtain an amount of smoothing that is reasonable and fully comparable for the non-stationary anisotropic and stationary isotropic method. This is apparent in Figures 1 and 2 that report the results obtained using SR-PDE and SSR in cases D and E. The top right panels show the comparison of the two methods in terms of RMSE of the corresponding estimators, evaluated on a fine lattice of step 0.01 over the whole domain Ω. The bottom left and right panels of these figures display the surface estimates obtained using respectively SR-PDE and SSR, analogously to Figures 8-10 of the main paper.

Comparing the results obtained with SR-PDE and SSR we can notice that the inclusion of the

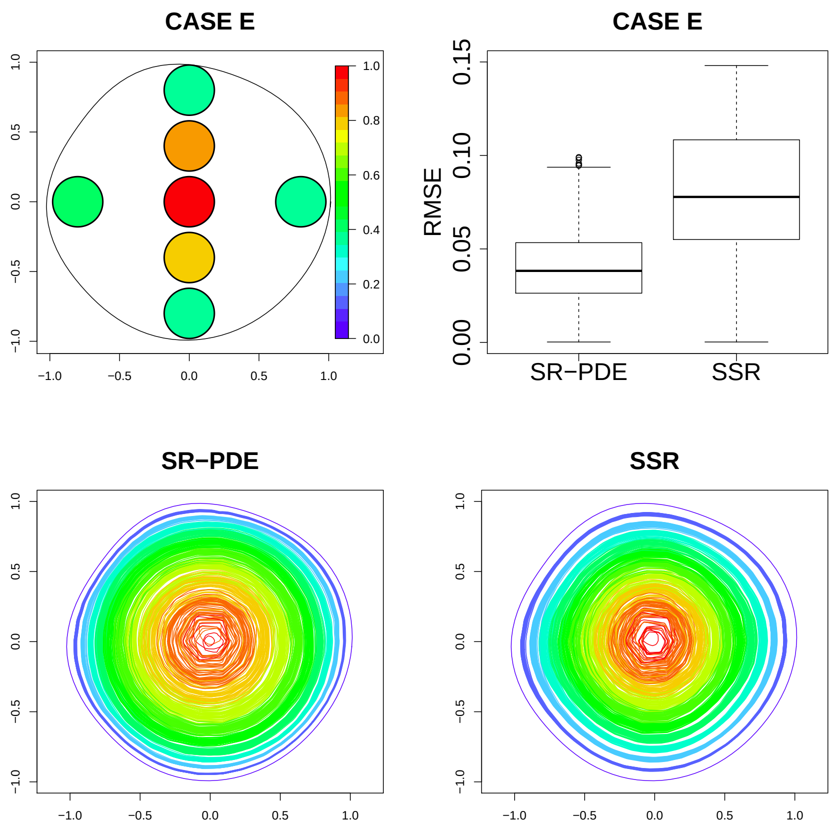

Figure 2: Same as Figure 1, for case E.

prior knowledge improves the estimate, especially when data are distributed only on subregions of the domain. In both the cases D and E, and especially in the latter, the boxplots highlight that SR-PDE smoothing provides significantly better estimates of f_0 than SSR. Similarly to the pointwise case C presented in the main paper, in the case E here considered SSR provides indeed surface estimates that depend on the cross-shape design of the experiments with isolines similar to rhomboids, instead of circles.